Phenomenology of Nuclear Shadowing in Deep-Inelastic Scattering[*]

G. Piller[a,c], W. Ratzka[b] and W. Weise[c]

[a]Department of Physics and Mathematical Physics, University of Adelaide,
S.A. 5005, Australia
[b]Institut für Theoretische Physik, Universität Regensburg,
D-93040 Regensburg, Germany
[c] Physik Department, Technische Universität München,
D-85747 Garching, Germany

Abstract

We investigate shadowing effects in deep-inelastic scattering from nuclei at small values $x < 0.1$ of the Bjorken variable. Unifying aspects of generalized vector meson dominance and color transparency we first develop a model for deep-inelastic scattering from free nucleons at small x. In application to nuclear targets we find that the coherent interaction of quark-antiquark fluctuations with nucleons in a nucleus leads to the observed shadowing at $x < 0.1$. We compare our results with most of the recent data for a large variety of nuclei and examine in particular the Q^2 dependence of the shadowing effect. While the coherent interaction of low mass vector mesons causes a major part of the shadowing observed in the Q^2 range of current experiments, the coherent scattering of continuum quark-antiquark pairs is also important and guarantees a very weak overall Q^2 dependence of the effect. We also discuss shadowing in deuterium and its implications for the quark flavor structure of nucleons. Finally we comment on shadowing effects in high-energy photon-nucleus reactions with real photons.

To be published in Z. Phys. A.

1 Introduction

In recent years numerous experiments have been dedicated to high precision measurements of deep-inelastic lepton scattering from nuclei. Experiments at CERN [1–4] and Fermilab [5–8] focus especially on the region of small values of the Bjorken variable $x = Q^2/2M\nu$, where $Q^2 = -q^2$ is the squared four-momentum transfer, ν the energy transfer and M the nucleon mass. The data, taken over a wide kinematic range from $10^{-5} < x < 0.65$ and $0.01\,\mathrm{GeV}^2 < Q^2 < 100\,\mathrm{GeV}^2$, show a systematic reduction of the nuclear structure function $F_2^A(x, Q^2)$ with respect to A times the free nucleon structure function $F_2^N(x, Q^2)$ at $x < 0.1$.

This so-called shadowing effect has prompted a fair amount of theoretical actvity (for recent reviews see e.g. [9]). Some of the existing work focuses on an infinite momentum frame description of the scattering process (see e.g. [10]). The driving mechanism in these models is given by quark and gluon annihilation processes at high parton densities, which are described using perturbative techniques. Although these methods allow one to address the Q^2 dependence of the shadowing effect, its x dependence is not accessible to perturbation theory and therefore subject to parametrization.

Another class of models considers the deep-inelastic scattering process in the laboratory frame where the target is at rest [11–20]. In this frame the interaction at small values of x proceeds via hadronic components present in the wave function of the exchanged virtual photon. The coherence length of these hadronic fluctuations is typically of order $1/Mx$ and exceeds, for $x < 0.1$, the average nucleon-nucleon distance in nuclei. Hence for small x the hadronic fluctuations will interact coherently with several nucleons inside the target nucleus. Shadowing is then caused by destructive interference of multiple scattering amplitudes which describe the passage of these fluctuations through the nucleus.

In this paper we present a laboratory frame description of deep-inelastic scattering at small x, based on ideas which unify the generalized vector meson dominance picture [21] with the concept of color transparency (for recent reviews see [22]). At small momentum transfers, $Q^2 < 1\,\mathrm{GeV}^2$, the hadronic components of the absorbed virtual photon are formed by strongly correlated quark-antiquark pairs, most prominently by the low mass vector mesons ρ, ω and ϕ. At larger Q^2 quark-antiquark pairs from the so-called $q\bar{q}$-continuum become increasingly important. We include both strongly correlated and continuum $q\bar{q}$-fluctuations in terms of the measured photon spectral function. While some empirical information is available about the interaction of low mass vector mesons with nucleons and nuclei, the interaction properties of continuum quark-antiquark pairs are scarcely known. To fill this gap we use color transparency as a guiding principle, i.e. the cross section of color singlet quark-antiquark pairs is assumed to be proportional to their transverse size. With these ingredients we obtain a good description of both nucleon and nuclear structure functions, $F_2^N(x, Q^2)$ and $F_2^A(x, Q^2)$, at small x.

Although some of the ideas mentioned above are common to several recent models of deep-inelastic scattering at small x, little effort has been directed towards a quantitative comparison with the now available large amount of experimental data. We confront our model with most of the recent data. In particular we examine the Q^2 dependence of the shadowing effect – an intensely discussed issue. We find that while a major part of the shadowing seen in current experiments is caused by the coherent interaction of low mass vector mesons, coherent scattering of continuum $q\bar{q}$ pairs is also important and guarantees a weak Q^2 dependence of the shadowing effect.

We will also briefly discuss shadowing effects in high-energy photon-nucleus reactions with real photons (i.e. in the limit $Q^2 \to 0$).

The plan of this paper is as follows: In Section 2 we introduce the space-time picture of deep-inelastic scattering in the laboratory frame. First we develop a model for deep-inelastic scattering from free nucleons in Section 3. Its extension to nuclear targets is described in Section 4. We discuss shadowing in nuclei with intermediate and large masses as well as its implications for a deuterium target. In Section 5 we apply our model to high energy photon-nucleus scattering. Finally, Section 6 contains a summary and conclusions.

2 Lab frame picture of deep-inelastic scattering

It is common to discuss deep-inelastic lepton scattering on free nucleons in a frame where the target moves with a large momentum, $|\mathbf{p}| \to \infty$. In this *infinite momentum frame* the parton model allows the interpretation of measured structure functions as momentum distributions of quarks and antiquarks in the target. There is, however, no reliable approach for dealing with nuclear systems in this frame. Consequently the preferable frame of reference for an investigation of nuclear effects in deep-inelastic scattering is rather the *laboratory system* in which the target is at rest. Well established knowledge about the structure and geometry of nuclear targets can then be used.

Consider therefore a description of deep-inelastic lepton scattering from nucleons or nuclei in the laboratory frame. Here the basic photon-nucleon interaction process involves the time orderings shown in Figs. 1(a) and 1(b): the photon either hits a quark (or antiquark) in the target which picks up the large energy and momentum transfer, or the photon converts into a quark-antiquark pair that subsequently interacts with the target.

For small x the pair production process (b) dominates the scattering amplitude, as can be shown in time-ordered perturbation theory [13]: The amplitudes $\mathcal{A}_a$ and $\mathcal{A}_b$ of processes (a) and (b) are roughly proportional to the inverse of their corresponding energy denominators ΔE_a and ΔE_b. For large energy trans-

2

fers $\nu \gg M$ these are:

$$\Delta E_a = E_a(t_2) - E_a(t_1) \approx -\left\langle p_q^2 \right\rangle^{1/2} + \frac{\frac{2}{3}\left\langle p_q^2 \right\rangle + Q^2}{2\nu}, \tag{1}$$

$$\Delta E_b = E_b(t_2) - E_b(t_1) \approx \frac{\mu^2 + Q^2}{2\nu}, \tag{2}$$

where $\left\langle p_q^2 \right\rangle^{1/2}$ is the average quark momentum in a nucleon and μ is the invariant mass of the quark-antiquark pair. We then obtain for the ratio of these amplitudes:

$$\left| \frac{\mathcal{A}_a}{\mathcal{A}_b} \right| \sim \left| \frac{\Delta E_b}{\Delta E_a} \right| \approx \frac{M x}{\left\langle p_q^2 \right\rangle^{1/2}} \left(1 + \frac{\mu^2}{Q^2} \right). \tag{3}$$

As we will argue later, the main contribution to process (b) comes from quark-antiquark pairs with a squared mass $\mu^2 \sim Q^2$. The ratio in Eq. (3) is evidently small compared to unity for $x \ll 0.1$. Hence pair production, Fig. 1(b), will be the dominant lab frame process in the small-x region. For the following discussion of deep-inelastic scattering at small x we will therefore consider the dominant process (b) only (although it should of course be noted that in principle only the sum of (a) and (b) is Lorentz and gauge invariant).

What are the implications of this picture for deep-inelastic scattering from nuclear targets? The coherence length λ of a photon-induced hadronic fluctuation with mass μ is given by the inverse of the energy denominator (2):

$$\lambda \sim \frac{1}{\Delta E_b} = \frac{2\nu}{\mu^2 + Q^2} \xrightarrow{\mu^2 \sim Q^2} \frac{1}{2xM}. \tag{4}$$

For $x < 0.05$ this coherence length exceeds the average distance between nucleons in nuclei, $d \approx 1.8\,\mathrm{fm}$. Then coherent multiple scattering on several nucleons in the target can occur, leading to nuclear shadowing.

For larger x, the coherence length of the intermediate $q\bar{q}$ state is small, $\lambda < d$. At the same time the process in Fig. 1(a) becomes prominent, i.e. the virtual photon is absorbed directly by a quark or antiquark in the target. In the range of intermediate and large x, say $x > 0.2$, the virtual photon therefore interacts incoherently with nucleons bound in the nuclear target.

3   Deep-inelastic scattering from free nucleons at small x

Before discussing effects in the scattering from nuclei we have to develop a model of deep-inelastic scattering from free nucleons. The free nucleon structure function $F_2^N(x, Q^2)$, defined as the average of the proton and the neutron structure function, can be written in terms of the virtual photon-nucleon cross section $\sigma_{\gamma^* N}$:

$$F_2^N(x, Q^2) = \frac{1-x}{1 + \frac{Q^2}{\nu^2}} \frac{Q^2}{4\pi^2 \alpha_{\mathrm{em}}} \sigma_{\gamma^* N}. \tag{5}$$

In the limit $\nu^2 \gg Q^2$ and $x < 0.1$ that we are concerned with, this simplifies to

$$F_2^N(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \sigma_{\gamma^* N} \tag{6}$$

(we use $\alpha_{\text{em}} \equiv e^2/4\pi = 1/137$). We will consider only contributions from transversely polarized photons, as they constitute the dominant part of the cross section: $\sigma_{\gamma^* N} \approx \sigma_{\gamma^* N}^T$ (see [23] for an experimental analysis of σ_L/σ_T).

As discussed above, the virtual photon interacts with the nucleon by first converting into a $q\bar{q}$ pair which then propagates, forming a hadronic intermediate state that interacts strongly with the nucleon. This is expressed in the following spectral ansatz for the structure function [11, 14, 20, 24, 25] valid at $x < 0.1$:

$$F_2^N(x, Q^2) = \frac{Q^2}{\pi} \int_{4m_\pi^2}^{\infty} d\mu^2 \frac{\mu^2 \, \Pi(\mu^2)}{(\mu^2 + Q^2)^2} \, \sigma_{hN}(\mu^2). \tag{7}$$

Here $\Pi(\mu^2)$ is the spectrum of hadronic fluctuations with mass μ which is related to the measured cross section for $e^+ e^- \to$ hadrons by

$$\Pi(s) = \frac{1}{12\pi^2} \frac{\sigma_{e^+e^- \to \text{hadrons}}(s)}{\sigma_{e^+e^- \to \mu^+\mu^-}(s)}. \tag{8}$$

Note that the effective hadron-nucleon cross section $\sigma_{hN}(\mu^2)$ in Eq. (7) is an average including all contributions with a given invariant mass μ. The factor $(\mu^2 + Q^2)^{-2}$ in Eq. (7) comes from the propagators of the hadronic intermediate states. One should of course note that Eq. (7) cannot be expected to follow directly from perturbative QCD. In particular, the effective cross section $\sigma_{hN}(\mu^2)$ incorporates non-perturbative physics characteristic of the small-x region. However, as will be shown, Eq. (7) does have the proper logarithmic behavior at large Q^2.

The structure function $F_2^N(x, Q^2)$ in Eq. (7) is dominated by contributions from intermediate states with an invariant mass $\mu^2 \sim Q^2$. As a consequence for small momentum transfer, $Q^2 < 1\,\text{GeV}^2$, the low mass vector mesons ρ, ω and ϕ are of major importance. They represent strongly correlated quark-antiquark pairs which contribute the term

$$\Pi^{\text{VMD}}(q^2) = \sum_{V=\rho,\omega,\phi} \left(\frac{m_V}{g_V}\right)^2 \delta(q^2 - m_V^2) \tag{9}$$

to the photon spectral function. Here m_V are the vector meson masses and g_V^{-1} the corresponding γV coupling constants (see Table 1). Eq. (9) represents the traditional Vector Meson Dominance (VMD) model. For large Q^2 the heavy vector mesons J/ψ and ψ' also contribute, and we take them into account as well. Altogether, vector mesons give a contribution to the nucleon structure function of the form

$$F_2^{N,\text{VMD}}(x, Q^2) = \frac{Q^2}{\pi} \sum_V \left(\frac{m_V^2}{g_V}\right)^2 \left(\frac{1}{m_V^2 + Q^2}\right)^2 \sigma_{VN}. \tag{10}$$

4

Here σ_{VN} are the vector meson-nucleon cross sections. They can be determined in real and virtual photoproduction experiments (see Table 1). It should be mentioned that their exact value may in principle depend on the kinematics of the experiment (see e.g. [29]).

For small values of x and Q^2 ($x < 0.1$ and $Q^2 < 1\,\mathrm{GeV}^2$) the interactions of the low mass vector mesons dominate the nucleon structure function F_2^N and lead to the scale breaking behavior $F_2^N(x, Q^2) \sim Q^2$ for $Q^2 \to 0$.

For larger values of the momentum transfer, i.e. $Q^2 > m_\phi^2 \approx 1\,\mathrm{GeV}^2$, the nucleon structure function F_2^N is governed by the interaction of quark-antiquark pairs with mass $\mu^2 \sim Q^2 > 1\,\mathrm{GeV}^2$. Apart from the narrow charmonium and upsilon resonances, these quark pairs form the so called $q\bar{q}$ continuum. In the annihilation of e^+e^- into hadrons they are responsible for the approximately constant behavior of the cross section ratio at large timelike momenta, $\sigma_{e^+e^- \to \mathrm{hadrons}}/\sigma_{e^+e^- \to \mu^+\mu^-} \approx 3\sum_f e_f^2$, where we sum over the fractional charges e_f of all quark flavors which are energetically accessible.

To calculate the contribution of the continuum quark-antiquark fluctuations to the nucleon structure function we need to know their effective interaction cross section. Since the $q\bar{q}$ fluctuations of the photon are color singlets, we assume their cross sections to scale with their transverse size ρ (i.e. their size in a plane perpendicular to their momentum) as $\sigma \sim \rho^2$ [16]. Investigating the geometry of the dissociation of a photon into an uncorrelated $q\bar{q}$ pair more closely (see e.g. [30]) one obtains the following approximate expression for ρ^2:

$$\rho^2 \approx \frac{1}{\alpha(1-\alpha)} \frac{4\mu^2}{(\mu^2 + Q^2)^2}. \tag{11}$$

Here α is the fraction of the light-cone momentum carried by the quark: if k_q is the quark momentum and $q = (q_0, \mathbf{0}_\perp, q_3)$ the photon momentum, one has $\alpha = (k_{q0} + k_{q3})/(q_0 + q_3)$. (Correspondingly, $1 - \alpha$ is the light-cone momentum fraction carried by the antiquark.) Of course Eq. (11) is a reasonable estimate for the size of the $q\bar{q}$ fluctuation only as long as the distance ρ of the pair is smaller than a typical confinement scale of about 1 fm. If the distance ρ increases further, strong interactions between the quark and antiquark will limit the transverse size of the $q\bar{q}$ fluctuation, thus leading to a saturation of ρ. Having this in mind, we choose for the effective cross section of continuum quark-antiquark pairs:

$$\sigma_{hN}(\mu^2, \alpha) = K \cdot \rho^2 = K \cdot \min \begin{cases} R_c^2 \\ \dfrac{1}{\alpha(1-\alpha)} \dfrac{4\mu^2}{(\mu^2 + Q^2)^2}, \end{cases} \tag{12}$$

with a constant K to be determined. Here we have introduced a maximum radius R_c which should be in the range of the confinement scale. As it is clear from our discussion above, the cross section $\sigma_{hN}(\mu^2, \alpha)$ depends not only on the invariant mass μ of the $q\bar{q}$ pair, but also on α, i.e. on the way the photon momentum is split between the quark and antiquark. From Eq. (12) we

observe that the average interaction cross section of $q\bar{q}$ pairs with mass μ is $\sigma_{hN}(\mu^2) = \int_0^1 d\alpha\,\sigma(\mu^2,\alpha) \sim 1/\mu^2$ (ignoring terms $\sim \log\mu^2$), which is the behavior necessary for scaling [14, 16, 20, 25].

If we now take into account both the vector mesons and the quark-antiquark continuum, we obtain from Eqs. (7, 10, 12) the following expression for the nucleon structure function:

$$F_2^N(x, Q^2) = \frac{Q^2}{\pi} \sum_{V=\rho,\dots} \left(\frac{m_V^2}{g_V}\right)^2 \left(\frac{1}{m_V^2 + Q^2}\right)^2 \sigma_{VN}$$
$$+ \frac{Q^2}{\pi} \int_{\mu_0^2}^\infty d\mu^2 \int_0^1 d\alpha \frac{\mu^2\,\Pi^{\text{cont.}}(\mu^2)}{(\mu^2 + Q^2)^2}\,\sigma_{hN}(\mu^2,\alpha), \tag{13}$$

valid at small Bjorken x. Here $\Pi^{\text{cont.}} = \Pi - \Pi^{\text{VMD}}$ is the continuum part of the photon spectral function which starts at $\mu_0^2 \sim m_\phi^2$. While the vector meson part vanishes as $1/Q^2$ for large Q^2, the $q\bar{q}$ continuum contribution to the structure function displays logarithmic scaling behavior:

$$F_2^N(x, Q^2) \sim \ln\left(R_c^2 Q^2\right) \quad \text{for } Q^2 \gg 1\,\text{GeV}^2. \tag{14}$$

We now compare our result for the free nucleon structure function F_2^N with recent data of the New Muon Collaboration [2]. We include in Eq. (13) all vector mesons ρ, ω, ϕ, J/ψ and ψ'. Their masses, coupling constants and cross sections are summarized in Table 1.

The effective cross section σ_{VN} (i.e. the forward scattering amplitude) may depend on the momentum and energy transfer variables Q^2 and ν. The relevant range in Q^2 is, however, restricted by the fact that the vector mesons contribute mainly in the region $Q^2 \approx m_V^2$. On the other hand experimental constraints [1,2] put bounds on the accessible values of ν. We therefore chose the cross sections to be approximately constant, setting $\sigma_{\rho N} = 22\,\text{mb}$ and fixing the other cross sections to scale like $\sigma_{VN} \sim 1/m_V^2$.

The constant K in Eq. (12) is fixed at $K = 1.7$ together with $R_c = 1.3\,\text{fm}$. This corresponds to a maximum value of about $29\,\text{mb}$ for the effective cross section of a $q\bar{q}$-pair interacting with a nucleon.

From Fig. 2 one can see that our model reproduces the measured nucleon structure function at small x quite well. We want to emphasize again the importance of vector mesons at small values of Q^2. In detail we find that at $Q^2 = 1\,\text{GeV}^2$ almost half of F_2^N at $x = 0.01$ is due to vector mesons. At $Q^2 = 10\,\text{GeV}^2$ they still contribute around 15%.

4   Deep-inelastic scattering from nuclei at small x

Just like the scattering from free nucleons, scattering from nuclear targets at small values of x proceeds via the interaction of hadronic components present in

the spectral function of the exchanged photon. For $x < 0.1$ the nuclear structure function F_2^A can therefore be written in a way analogous to F_2^N in Eq. (13):

$$F_2^A(x, Q^2) = \frac{Q^2}{\pi} \sum_{V=\rho,\dots} \left(\frac{m_V^2}{g_V}\right)^2 \left(\frac{1}{m_V^2 + Q^2}\right)^2 \sigma_{VA}$$
$$+ \frac{Q^2}{\pi} \int_{\mu_0^2}^{\infty} d\mu^2 \int_0^1 d\alpha \frac{\mu^2 \Pi^{\text{cont.}}(\mu^2)}{(\mu^2 + Q^2)^2} \sigma_{hA}(\mu^2, \alpha). \tag{15}$$

We have just replaced the hadron-nucleon cross sections σ_{hN} in Eq. (13) by the corresponding hadron-nucleus cross sections σ_{hA}.

As mentioned in Section 2, for $x < 0.05$ the coherence length $\lambda = 2\nu/(Q^2 + \mu^2)$ of the interacting hadronic fluctuation exceeds the average internucleon distance in nuclei. Consequently the intermediate hadronic system can scatter coherently from several nucleons in the target. Interference between the multiple scattering amplitudes causes a reduction of the hadron-nucleus cross sections compared to the naïve result of just A times the respective hadron-nucleon cross section and thus leads to shadowing.

These effects are described by the Glauber-Gribov multiple scattering formalism [31] which we will now summarize briefly.

4.1 Glauber-Gribov multiple scattering theory

Let us consider high energy forward scattering of a hadronic fluctuation h with four-momentum $q = (\nu, \mathbf{0}_\perp, \sqrt{\nu^2 + Q^2})$ and mass μ on a nucleus. In the laboratory frame the target momentum is $P = (A(M - \mathcal{E}), \mathbf{0})$, where A is the nuclear mass number and $\mathcal{E}$ the binding energy per nucleon. The scattering amplitude $\mathcal{A}_{hA}$ for this process can be written as the sum $\mathcal{A}_{hA} = \sum_{n=1}^{A} \mathcal{A}_h^{(n)}$ over multiple scattering terms $\mathcal{A}_h^{(n)}$, each of which describes the projectile interacting consecutively with n nucleons in the target (see Fig. 3):

$$\mathcal{A}_h^{(n)} = \frac{A!}{(A-n)!} \prod_{i=1}^{n-1} \left[\int \frac{dl_i}{(2\pi)2(M-\mathcal{E})} \right] V_h^{(n)}(\nu, \dots q_{iz} \dots)$$
$$\times \int d^2 b \int_{-\infty}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \cdots \int_{z_{n-1}}^{\infty} dz_n$$
$$\times \rho_n(\mathbf{b}, z_1 \dots z_n) \prod_{i=1}^{n-1} \left[e^{il_i(z_i - z_{i+1})} \right]. \tag{16}$$

Here $V_h^{(n)}(\nu, \dots q_{iz} \dots)$ describes the interaction of the hadronic projectile with n nucleons. For large projectile energies ν, it is assumed that V depends only on ν and q_{iz}, the longitudinal momenta transferred to the interacting nucleons. Since we consider forward scattering only, we have $\sum_{i=1}^{n} q_{iz} = 0$. Furthermore the integration variables l_i are defined as $l_i = \sum_{j=1}^{i} q_{jz}$, such that $|\mathbf{q}| - l_i$ is the

longitudinal momentum of the projectile after its interaction with the ith nucleon. The nucleon distribution in Eq. (16) is given by the n-particle density

$$\rho_n(\mathbf{b}; z_1 \ldots z_n) = \prod_{j=n+1}^{A} \left[\int \mathrm{d}^3 x_j \right] \delta^3(\mathbf{X}_{\mathrm{cm}})$$

$$\times \left| \Psi \left(\mathbf{b}, z_1; \ldots; \mathbf{b}, z_n; \mathbf{x}_{n+1} \ldots \mathbf{x}_A \right) \right|^2, \tag{17}$$

where $\Psi(\ldots \mathbf{x}_i \ldots)$ is the coordinate-space wave function of the nucleus. Its center of mass $\mathbf{X}_{\mathrm{cm}} = \frac{1}{A} \sum_{i=1}^{A} \mathbf{x}_i$ is fixed at the origin. Since the high energy scattering process occurs at a fixed impact parameter $\mathbf{b}$, the active nucleons enter ρ_n at coordinates $\mathbf{x}_i = (\mathbf{b}, z_i)$ for $i = 1 \ldots n$.

As a next step the amplitude $V_h^{(n)}$ is expanded in hadronic eigenstates. Let us denote the complete set of states that can be reached after the interaction with the ith nucleon by $\{h_i\}$ and write the corresponding invariant masses as m_{h_i}. If the conversion from state h_i into state h_{i+1} in the interaction with the $(i+1)$th nucleon is described by the transition amplitude $f_{h_i h_{i+1}}$, the expression for $V_h^{(n)}$ becomes:

$$\begin{aligned} \mathrm{i} V_h^{(n)} &= \sum_{h_1, \ldots, h_{n-1}} \mathrm{i} f_{h h_1} \frac{\mathrm{i}}{\nu^2 - (|\mathbf{q}| - l_1)^2 - m_{h_1}^2 + \mathrm{i}\epsilon} \mathrm{i} f_{h_1 h_2} \\ &\times \frac{\mathrm{i}}{\nu^2 - (|\mathbf{q}| - l_2)^2 - m_{h_2}^2 + \mathrm{i}\epsilon} \mathrm{i} f_{h_2 h_3} \times \cdots \\ \cdots &\times \frac{\mathrm{i}}{\nu^2 - (|\mathbf{q}| - l_{n-1})^2 - m_{h_{n-1}}^2 + \mathrm{i}\epsilon} \mathrm{i} f_{h_{n-1} h}. \end{aligned} \tag{18}$$

We can now perform the integration over the variables l_i. We note that the exponential factors in Eq. (16) require that the integration contour be closed in the lower plane. Picking up the poles, the longitudinal momentum transfer gets fixed at

$$l_i = |\mathbf{q}| - \sqrt{\nu^2 - m_{h_i}^2} \approx \frac{Q^2 + m_{h_i}^2}{2\nu}. \tag{19}$$

For intermediate and heavy nuclei we may in good approximation consider only elastic rescattering of the incoming hadronic state h from the nucleons inside the target. Contributions of inelastically produced states to multiple scattering were investigated by Murthy et al. [32] and Nikolaev [33] for high energy hadron-nucleus scattering processes. They found such contributions to be small, though rising logarithmically with the projectile energy ν. For example at $\nu \sim 100\,\mathrm{GeV}$ inelastic terms account typically for $\sim 5\%$ of the total hadron-nucleus cross sections under consideration.

In the so-called "diagonal approximation", i.e. neglecting inelastic intermediate states, the amplitude $V_h^{(n)}$ reduces to:

$$\mathrm{i} V_{h\,\mathrm{diag.}}^{(n)} = \prod_{i=1}^{n-1} \left[\mathrm{i} f_{hh} \frac{\mathrm{i}}{\nu^2 - (|\mathbf{q}| - l_i)^2 - \mu^2 + \mathrm{i}\epsilon} \right] \mathrm{i} f_{hh}, \tag{20}$$

8

In this case the longitudinal momentum transfer is fixed just at the inverse of the coherence length λ (4) of the hadronic projectile:

$$l = |\mathbf{q}| - \sqrt{\nu^2 - \mu^2} \approx \frac{Q^2 + \mu^2}{2\nu} = 1/\lambda. \tag{21}$$

Summing over all multiple scattering terms $\mathcal{A}_h^{(n)}$ and neglecting the binding energy $\mathcal{E} \ll M$ we find for the hadron-nucleus forward scattering amplitude

$$
\begin{aligned}
\mathrm{i}\mathcal{A}_{hA} = & \sum_{n=1}^{A} \left\{ \frac{A!}{(A-n)!} (4M\nu)^{-n+1} (\mathrm{i}f_{hh})^n \right. \\
& \times \int \mathrm{d}^2b \int_{-\infty}^{\infty} \mathrm{d}z_1 \int_{z_1}^{\infty} \mathrm{d}z_2 \cdots \int_{z_{n-1}}^{\infty} \mathrm{d}z_n \\
& \left. \times \rho_n(\mathbf{b}; z_1 \ldots z_n) \exp\left(\mathrm{i}\frac{z_1 - z_n}{\lambda}\right) \right\}.
\end{aligned}
\tag{22}
$$

With the assumption that the hadronic forward amplitudes f_{hh} are dominated by their imaginary parts (see [34]), we can use the optical theorem to replace

$$\mathrm{i}f_{hh} \approx -2M\nu\sigma_{hN}. \tag{23}$$

We finally obtain the following expression for the hadron-nucleus cross section:

$$
\begin{aligned}
\sigma_{hA} = & \sum_{n=1}^{A} \left\{ \frac{A!}{(A-n)!} \left(-\frac{1}{2}\right)^{n-1} (\sigma_{hN})^n \right. \\
& \times \mathrm{Re}\left[\int \mathrm{d}^2b \int_{-\infty}^{\infty} \mathrm{d}z_1 \int_{z_1}^{\infty} \mathrm{d}z_2 \cdots \int_{z_{n-1}}^{\infty} \mathrm{d}z_n \right. \\
& \left.\left. \times \rho_n(\mathbf{b}; z_1 \ldots z_n) \exp\left(\mathrm{i}\frac{z_1 - z_n}{\lambda}\right) \right] \right\} \\
= & A\sigma_{hN} \left(1 + \sum_{n=2}^{A} (-1)^{n-1} C_n \left(\sigma_{hN}\right)^{n-1} \right),
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
C_n = & \frac{(A-1)!}{2^{n-1}(A-n)!} \mathrm{Re}\left[\int \mathrm{d}^2b \int_{-\infty}^{\infty} \mathrm{d}z_1 \int_{z_1}^{\infty} \mathrm{d}z_2 \cdots \int_{z_{n-1}}^{\infty} \mathrm{d}z_n \right. \\
& \left. \times \rho_n(\mathbf{b}; z_1 \ldots z_n) \exp\left(\mathrm{i}\frac{z_1 - z_n}{\lambda}\right) \right].
\end{aligned}
\tag{25}
$$

Note that the exponential factor in Eq. (24) oscillates rapidly if the coherence length λ of the hadronic scatterer is small. In that case all terms in the series with $n > 1$ approximately vanish and one finds $\sigma_{hA} \approx A\sigma_{hN}$. In the small-$x$ region, however, λ increases and higher order terms contribute, leading to a reduction of σ_{hA}.

Let us take a closer look at the multiple scattering series (24). For $n \ll A$ we may neglect the recoil motion of the $A - n$ noninteracting nucleons. In the absence of nuclear correlations the n-particle density is then approximated by:

$$\rho_n(\mathbf{b}; z_1 \ldots z_n) \approx \frac{1}{A^n} \prod_{i=1}^{n} \rho(\mathbf{b}, z_i), \tag{26}$$

where ρ is the nuclear one-body density, normalized as $\int \mathrm{d}^3 x \rho(\mathbf{x}) = A$. For light nuclei only single and double scattering contributions are of importance. The above approximation may already be applied for $A \geq 6$. Furthermore the validity of (26) improves with increasing A, since the number of rescatterings n_{eff} that add significantly to σ_{hA} grows at most as the nuclear diameter, i.e. $n_{\mathrm{eff}} \sim A^{1/3}$.

For illustration, consider the multiple scattering series (24) with a Gaussian density ρ of radius $\langle r^2 \rangle^{1/2} = \sqrt{3/2} \, a A^{1/3}$:

$$\rho(\mathbf{r}) = \frac{1}{\pi^{3/2} a^3} \exp\left(-\frac{\mathbf{r}^2}{a^2 A^{2/3}}\right). \tag{27}$$

We obtain:

$$\sigma_{hA} \approx A \sigma_{hN} \left[1 - A^{1/3} \frac{\sigma_{hN}}{8\pi a^2} \frac{A-1}{A} \exp\left(-\frac{a^2 A^{2/3}}{2\lambda^2}\right) + \ldots\right]. \tag{28}$$

We observe that the double scattering contribution adds to the single scattering term a negative correction, the magnitude of which grows as $A^{1/3}$. Furthermore we notice that the shadowing correction decreases rapidly if the coherence length of the scatterer becomes small, $\lambda < \langle r^2 \rangle^{1/2}$.

4.2 Shadowing in intermediate-mass and heavy nuclei

In the previous section we have prepared the tools to calculate total hadron-nucleus cross sections σ_{hA} from the respective hadron-nucleon cross sections σ_{hN}. We can now proceed to calculate the nuclear structure function as given by Eq. (15).

We will first discuss heavier nuclei, making use of the approximation in Eq. (26), i.e. replacing the n-particle density ρ_n by a product of one-body densities. We use two 'extreme' parametrizations for these nuclear matter densities: a Gaussian shape as in Eq. (27) for small A and a square well shape for heavier nuclei. In both cases we fit the mean square radii of these density distributions to empirical nuclear radii [35]. Note that in earlier calculations [20] we have used realistic densities and included two-nucleon correlations, but we found the resulting corrections in both cases to be systematically very small. With this as an input, we can now calculate the shadowing ratios

$$R(x, Q^2) = \frac{F_2^A(x, Q^2)}{A \, F_2^N(x, Q^2)}. \tag{29}$$

Figures 4 and 5 show our calculated ratios for various nuclei, together with experimental results obtained by the NMC at CERN [1,4] and the E-665 collaboration at FNAL [5–7] who have performed muon scattering measurements focusing on the small-x region. We see that for $x < 0.1$ the ratio (29) is generally below one, i.e. shadowing occurs. In this x range we can apply the physical picture introduced in Section 2: The virtual photon interacts with the target nucleus through hadronic fluctuations. For small x the coherence length λ of the fluctuations becomes large enough to make multiple scattering processes contribute significantly.

However the shadowing ratio (29) is not just a function of x but also depends (weakly) on Q^2. We recall from our previous discussion that the value of Q^2 basically selects that part of the hadron mass spectrum which dominates the interaction, and hence determines which cross sections σ_{hN} contribute significantly to the multiple scattering series. From Section 3 we note that σ_{hN} depends not only on the mass μ of the $q\bar{q}$ pair, but also on the distribution of momenta within that pair. While the averaged interaction cross sections decrease as $\log(\mu^2)/\mu^2$ with increasing mass, pairs which are asymmetric in the $q\bar{q}$ phase space interact with large cross sections, even for large μ, and therefore produce strong shadowing. This is the reason for the very weak overall Q^2 dependence of the shadowing effect.

The relevant experiments all operate on fixed targets within a limited range of muon energies, hence Q^2 is not an independent parameter but depends on the x-range considered. We have taken this dependence into account by inserting into our calculation the mean Q^2 values reported for the different x-bins of the experiments. With decreasing x the accessible values for Q^2 also become small (e.g. at $x = 0.005$, $Q^2 \sim 1\,\mathrm{GeV}^2$ for the NMC experiment from ref. [1]). Therefore the contributions of the low mass vector mesons ρ, ω and ϕ dominate the observed shadowing at $x < 0.01$ as indicated in Figure 4 and 5.

The NMC [1,4] has analyzed the Q^2 dependence of shadowing by performing linear fits $R(x, Q^2) \approx a + b \ln Q^2$ to the data for every x-bin. Fig. 6 shows the slopes b so obtained in comparison with our calculations. We see that the NMC data are compatible with basically no Q^2 dependence. Our calculations give a very small positive slope, i.e. a slow decrease in shadowing with increasing Q^2 which is within the range of the NMC data.

An NMC analysis of the structure function ratio Sn/C is underway. It combines data taken at several different muon energies and provides considerably better statistics. Figure 7 shows our predictions for this ratio in about the kinematic region to be covered.

Both the E-665 data on Xenon [5] and the recent NMC data for Carbon and Lithium [4] extend to rather small values of x ($x < 10^{-3}$). In this region a saturation of the shadowing effect becomes apparent, with the ratio eventually approaching the 'photon point' i.e. the value observed in the scattering of real photons on nuclei (see Section 5 below). Figures 8 a) and b) display the shadowing

11

ratio for Xenon, computed at various fixed values of the energy transfer ν, as a function of x and a function of Q^2, respectively. While the onset of shadowing is controlled by the coherence length λ, which enters as a function of x, one sees that the relevant variable for the saturation is Q^2. As we have already argued, variation of Q^2 basically scans the hadronic mass spectrum of the photon. Due to the experimental constraints mentioned above, small x in practice always implies small Q^2. Saturation occurs at values of Q^2 less than m_ρ^2, where the interaction is dominated by multiple scattering of the ρ meson.

Here a remark is in order about contributions from inelastic intermediate states to the multiple scattering series (which we have neglected). These are significant only at very small $x < 10^{-3}$ and turn out to be small for heavy nuclei. Their major contribution to shadowing increases logarithmically with the energy transfer ν but is independent of Q^2 at small $Q^2 < 1\,\mathrm{GeV}^2$ (see Section 4.3 and ref. [36]). Although the saturation value of $R(x, Q^2)$ at $x \ll 0.1$ may therefore depend on ν, the onset of the saturation is still controlled by Q^2.

Figure 9 shows the shadowing ratio $12 F_2^A / A F_2^{\mathrm{C}}$ for different nuclei plotted against $\log A$ at several values of x, together with preliminary NMC data [37]. One sees that the dependence on A is much weaker than the behavior $\sim A^{1/3}$ one would derive by only considering the double scattering term in (28). In fact for heavier nuclei, higher order contributions in the multiple scattering series become important and partly cancel the effect of the double scattering term. The result is a much less pronounced A dependence that can be fitted by the expression

$$12 F_2^A / A F_2^{\mathrm{C}} \approx a_x + b_x \ln A \,. \tag{30}$$

Figure 10 displays the slopes b_x in the different x bins resulting from our calculation and those extracted in the preliminary NMC analysis.

Note, however, that the behavior according to Eq. (30) cannot be correct in the limit of large A. One should rather expect saturation of the shadowing in this region, a hint of which can be seen in our results.

On the whole, our model is able to reproduce the shadowing phenomena in deep-inelastic scattering on heavy nuclei remarkably well. With respect to the Q^2 dependence, the pending release of the NMC data on Sn may be interesting.

4.3   Shadowing effects in deuterium

Shadowing also occurs in deuterium, the most weakly bound nucleus. Although small, this effect is of special interest since deep-inelastic scattering from deuterium is used to determine the structure function of the neutron. With the assumption of isospin symmetry, the proton and neutron structure functions together reveal information on the quark flavor structure of the nucleon. These reasons and recent high precision measurements of proton and deuteron structure functions and their ratio [2, 3, 8] inspired a lively activity on this topic (see e.g. [17, 38]).

Following our previous discussions, we now calculate the deuteron structure function F_2^D at small values of x, taking shadowing corrections explicitly into account. Consequences for the experimental extraction of the neutron structure function F_2^n will then be outlined briefly.

To calculate F_2^D at $x < 0.1$ we again need to know the interaction cross section σ_{hD} for the scattering of a hadronic fluctuation from the deuteron target (see Eq. (15)). In addition to incoherent scattering from the two nucleons, σ_{hD} includes a coherent double scattering correction:

$$\sigma_{hD} = 2\,\sigma_{hN} - \delta\sigma_{hD}. \tag{31}$$

From the multiple scattering series in Eqs. (16,18) we find

$$\delta\sigma_{hD} = \frac{1}{2}\sum_X \frac{|f_{hX}|^2}{(2M\nu)^2} F_L\big(1/\lambda(m_X)\big). \tag{32}$$

In contrast to our treatment of multiple scattering corrections in heavy nuclei in Section 4.2, we now take inelastic intermediate states explicitly into account. The transition amplitude f_{hX} describes the interaction of the hadronic state h with a nucleon by which h is converted into a state X with mass m_X. The coherence length $\lambda(m_X)$ of the hadronic state X is defined as in Eq. (4); we explicitly note its dependence on the mass of the propagating state. This coherence length enters via the longitudinal form factor F_L of the deuteron, which can be written in terms of the deuteron wave function as follows:

$$F_L(1/\lambda) = \int \mathrm{d}z\, |\psi(\mathbf{0}_\perp, z)|^2 \exp\left(i\frac{z}{\lambda}\right), \tag{33}$$

$$\text{with}\quad \psi(\mathbf{r}) = \psi_{J=1,M_J}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\left\{\frac{u(r)}{r} + \frac{w(r)}{r}\frac{1}{\sqrt{8}}S_{12}(\hat{r})\right\}\chi_{1,M_J}. \tag{34}$$

Here $u(r)$ and $w(r)$ are the S- and D-wave components of the deuteron wave function and χ_{1,M_J} is its spin part. $S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})\,(\sigma_2 \cdot \hat{r}) - (\sigma_1 \cdot \sigma_2)$ with $\hat{r} = \mathbf{r}/|\mathbf{r}|$ is the tensor operator. After taking the average over the target spin we find

$$F_L(1/\lambda) = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{d}z}{z^2}\left[u^2(z) + w^2(z)\right]\cos\left(\frac{z}{\lambda}\right). \tag{35}$$

Our expression for the double scattering correction (32) can be split into an elastic ($X = h$) and an inelastic contribution ($X \neq h$). As in Section 4.1, we assume that the amplitudes f_{hX} are strongly peaked in forward direction and dominated by their imaginary parts. We may then identify the inelastic contribution with the cross section for inelastic diffractive dissociation in the forward direction, $h + N \to X + N$, and obtain:

$$\delta\sigma_{hD} = \frac{1}{2}\sigma_{hN}^2 F_L\big(1/\lambda(m_h = \mu)\big) + 8\pi \int \mathrm{d}m_X^2 \left.\frac{\mathrm{d}^2\sigma_{h\to X}^{\text{inel.}}}{\mathrm{d}m_X^2 dt}\right|_{t=0} F_L\big(1/\lambda(m_X)\big), \tag{36}$$

where t is the squared momentum transfer.

A well known feature of diffractive dissociation of hadrons and photons is the $1/m_X^2$ mass spectrum at large m_X (see e.g. [39]). For hadron h this reads

$$\frac{1}{\sigma_{hN}} \frac{\mathrm{d}^2\sigma_{h\to X}^{\text{inel.}}}{\mathrm{d}m_X^2 dt}\bigg|_{t=0} \approx \frac{C}{m_X^2} \quad \text{for } m_X^2 \gg \mu^2, \tag{37}$$

where the constant $C \approx 0.1\,\text{GeV}^{-2}$ can be extracted from the experimental analysis in refs. [40]. We will use Eq. (37) to estimate the inelastic contributions to the double scattering correction in Eq. (36), assuming that the $1/m_X^2$ behavior of the diffractive cross section sets in at $m_X^2 = (\mu + \Lambda)^2$. In high energy hadron-nucleon scattering experiments [40] one finds Λ typically to be of the order of $\Lambda \sim 1\,\text{GeV}$.

With Eqs. (15,31) we may now calculate the deuteron structure function F_2^D. We use a sample of different deuteron wave functions for this purpose: those obtained from the realistic Paris [41] and Bonn [42] nucleon-nucleon potentials, but also – just for comparison – the simple but unrealistic Hulthén ansatz [43]. We discuss our results for F_2^D as above in terms of the structure function ratio

$$R^D(x, Q^2) = \frac{F_2^D(x, Q^2)}{2F_2^N(x, Q^2)} = 1 - \frac{\delta F_2^D(x, Q^2)}{2F_2^N(x, Q^2)}. \tag{38}$$

In Fig. 11 we display $R^D(x, Q^2)$ as a function of x for different values of the momentum transfer Q^2. We observe that $R^D(x, Q^2) < 1$ in the range $x < 0.1$, i.e. the characteristic shadowing behavior. The magnitude of the effect is small but depends on the deuteron wave function used as an input. For example at $x = 0.01$ and $Q^2 = 4\,\text{GeV}^2$ the calculated shadowing effect varies between $(1$–$2)\%$ (it amounts to 4% for the naive Hulthén function).

This sensitivity is a consequence of significant differences, for different potentials, in the short distance behavior of the deuteron density $\rho(r) = (u^2(r) + w^2(r))/(4\pi r^2)$, which determines the longitudinal form factor F_L in Eq. (35). In Fig. 12 we present $\rho(r)$ for the various deuteron wave functions, with the densities differing considerably for $r < 1\,\text{fm}$. On the other hand we note that the region $r < 1\,\text{fm}$ strongly influences F_L for $\lambda > 2\,\text{fm}$ as can be seen from Eq. (35). As we have learned in 4.1, such values of the propagation length control the nuclear shadowing effect.

In Fig. 13 we compare the calculated shadowing effect with and without contributions from inelastic intermediate states at fixed $Q^2 = 4\,\text{GeV}^2$ for the Paris wave function. We observe that inelastic states are important only for $x < 5 \times 10^{-3}$. For example at $x = 10^{-3}$ they account for about $\sim 20\%$ of the total shadowing effect. Their contribution decreases logarithmically with increasing x or, equivalently, with decreasing photon energy ν.

We may now briefly justify our statement in section 4.1 that contributions from inelastic intermediate states in multiple scattering are small in heavier nuclei. In analogy to Eq. (32) these contributions are proportional to the longitudinal nuclear form factor of the nucleus which for the example of a Gaussian

14

density reads [36]

$$F_L\big(1/\lambda(m_X)\big) \sim \exp\left(-\frac{\langle r^2 \rangle}{3\lambda(m_X)^2}\right). \tag{39}$$

From Eq. (4) we know that the coherence lengths of intermediate states decrease with the invariant mass of the propagating states. Since the invariant mass of the hadronic projectile h is always smaller than the mass of the diffractively excited inelastic intermediate states, the elastic contribution will naturally dominate the multiple scattering process. This dominance is more pronounced as the radius $\langle r^2 \rangle^{1/2}$ of the nuclear target increases.

What are the consequences of the shadowing effect in deuterium? As already mentioned, the neutron structure function F_2^n is usually extracted from a comparison of the deuteron and the proton structure function. Such an analysis has been performed recently by the NMC experiment [3] which investigated the kinematic region $x < 0.1$ with high accuracy. In this analysis, however, effects from nuclear shadowing in deuterium have been ignored; the difference of proton and neutron structure function was obtained by simply taking

$$F_{2,\text{NMC}}^{p-n} \equiv (F_2^p - F_2^n)_{\text{NMC}} = 2F_2^p - F_2^D. \tag{40}$$

However, for small x shadowing must be taken into account. The true structure function should therefore read:

$$F_2^{p-n} \equiv F_2^p - F_2^n = 2F_2^p - (F_2^D + \delta F_2^D) = F_{2,\text{NMC}}^{p-n} - \delta F_2^D. \tag{41}$$

The shadowing correction δF_2^D reduces the result with respect to the quoted difference $F_{2,\text{NMC}}^{p-n}$ (and correspondingly the true neutron structure function should be larger than the value obtained by the NMC).

The full symbols in Fig. 14 display the original NMC data for the difference $F_{2,\text{NMC}}^{p-n}$ as well as corrected results with the shadowing term subtracted for small x. In our calculation we used $Q^2 = 4\,\text{GeV}^2$, as the NMC analysis operates with structure functions interpolated to this value, and again a set of different deuteron wave functions. We notice that for small x the structure function difference becomes small, so that the relative size of the correction is of the order of 100%. This is in good agreement with the expectation that any deviation of $F_{2,\text{NMC}}^{p-n}$ from zero in this x region should be mostly due to the shadowing effect.

Let us next consider the integral over the difference of proton and neutron structure functions:

$$I_G(x,1) = \int_x^1 \frac{\mathrm{d}x'}{x'} \left(F_2^p(x') - F_2^n(x')\right). \tag{42}$$

In the parton model I_G can be rewritten in terms of quark distributions:

$$I_G(x,1) = \frac{1}{3}\int_x^1 \mathrm{d}x'\,(u_v(x') - d_v(x')) + \frac{2}{3}\int_x^1 \mathrm{d}x'\left(\bar{u}(x') - \bar{d}(x')\right). \tag{43}$$

15

The up and down valence quark distributions $u_v = u - \bar{u}$ and $d_v = d - \bar{d}$ in the proton are given by the difference of the respective quark and antiquark distributions. To obtain Eq. (43) we have used isospin symmetry to relate proton and neutron quark distributions.

If the first moments of the up and down sea quark distributions are approximately equal – this is fulfilled trivially if one assumes the sea to be SU(2)-flavor symmetric – the second term in Eq. (43) vanishes for $x \to 0$, and one arrives at the Gottfried sum rule [44]:

$$I_G = I_G(0,1) = \frac{1}{3} \int_0^1 dx' \, (u_v(x') - d_v(x')) = \frac{1}{3}. \tag{44}$$

Without taking shadowing into account, the NMC found $I_G^{\text{exp.}}(0,1) = 0.235 \pm 0.026$ [3]. This includes contributions from the unmeasured regions $x > 0.8$ and $x < 0.004$. A smooth extrapolation of F_2^n/F_2^p for $x \to 1$ yields $I_G(0.8,1) = 0.001 \pm 0.001$, while within conventional Regge theory $I_G(0,0.004) = 0.013 \pm 0.005$ is found [3].

The deviation of the Gottfried sum $I_G^{\text{exp.}}$ from the naïve expectation $1/3$ has been the target of some activity, see e.g. [45].

Let us now consider the impact of our corrections for deuteron shadowing on the extraction of the Gottfried sum. As discussed above, for $x < 0.1$ they significantly reduce the difference F_2^{p-n} with respect to the values used by the NMC collaboration.

The open symbols in Fig. 14 show the NMC results for the integrals $I_G(x,0.8)$ together with our corrected values. In Tab. 2 we give the corrections $\Delta I_G(0.004, 0.1)$ due to shadowing obtained for different deuteron wave functions. They reduce the extracted value for the Gottfried sum by 10% or more. This sizable correction is due to the fact that the structure function difference in the integral in (42) is weighted by a factor $1/x$. We see that the shadowing corrections further enhance the deviation of the Gottfried sum from the naïve value $I_G = 1/3$.

The E665 collaboration has also measured the structure function ratio F_2^d/F_2^p, for which preliminary data are now available [8]. Their x-range extends down to 10^{-6}, with the average Q^2 strongly dependent on x ($\langle Q^2 \rangle = 0.002 \, \text{GeV}^2$ for the lowest x bin $10^{-6} < x < 10^{-5}$). Figure 15 shows the ratio $F_2^d/2F_2^p$ as obtained by the E665 group. The small-x behavior of these data is in excellent agreement with the results of our model calculation. In comparing with Figs. 11 and 13 we note that the logarithmic growth of shadowing for $x \to 0$ displayed there is a result of plotting the ratio at constant Q^2, which implies a photon energy $\sim 1/x$. In this experiment the muon (and hence the photon) energy is limited and therefore the contributions from inelastic intermediate states are bounded.

16

5 Nuclear shadowing of real photons at high energy

In deep-inelastic scattering, the lepton beam is a source of highly energetic *virtual* photons. We have conveniently written the structure function in terms of a total cross section for the interaction of the virtual photon with the target (see Eq. 5). In the limit $Q^2 \to 0$ this cross section becomes physical, describing real photoproduction processes.

While hadronic vacuum fluctuations decouple from the photon in the limit $Q^2 \to 0$, so as to keep the real photon massless, shadowing still occurs via the production of quark-antiquark pairs on nucleons and their subsequent propagation through the nucleus. The coherence length of such hadronic states of mass μ, produced by a real photon of energy ν, is

$$\lambda = \frac{2\nu}{\mu^2}. \tag{45}$$

This becomes largest for those states with the smallest mass. For the ρ meson with $\mu^2 = m_\rho^2$ we find that λ reaches typical internucleon distances for $\nu > 3\,\text{GeV}$. We therefore expect multiple scattering to reduce the cross section for nuclear photoproduction $\sigma_{\gamma A}$ with respect to A times the photoproduction cross section on free nucleons $\sigma_{\gamma N}$.

Let us discuss this more quantitatively. The expression for $\sigma_{\gamma N}$ at large ν is obtained from Eq. (13) taking the limit $Q^2 \to 0$. This yields

$$
\begin{aligned}
\sigma_{\gamma N} &= \left(\sigma_{\gamma N}\right)_{\text{VDM}} + \left(\sigma_{\gamma N}\right)_{\text{cont.}} \\
&= \sum_V \frac{4\pi\alpha_{\text{em}}}{g_V^2}\sigma_{VN} + 4\pi\alpha_{\text{em}}\int_{\mu_0^2}^{\infty}\mathrm{d}\mu^2\int_0^1\mathrm{d}\alpha\frac{\Pi^{\text{cont.}}(\mu^2)}{\mu^2}\sigma_{hN}(\mu^2,\alpha). \tag{46}
\end{aligned}
$$

The VMD term ($\sim 90\mu$b) is about twice as large as the continuum contribution.

As in the case of deep-inelastic scattering, we can calculate the cross section for photoproduction on nuclei by replacing the hadron-nucleon cross sections in Eq. (46) by the corresponding hadron-nucleus cross sections, obtained via the multiple scattering formalism as outlined in 4.1. To discuss nuclear effects it is common to consider the ratio

$$\frac{A_{\text{eff}}}{A} = \frac{\sigma_{\gamma A}}{A\,\sigma_{\gamma N}} \tag{47}$$

In our calculation of $\sigma_{\gamma N}$ we use parameters which were fixed through our fit to the nucleon structure function F_2^N in Section 3. To calculate $\sigma_{\gamma A}$ we employ similar nuclear densities as in the case of nuclear deep-inelastic scattering in Sec. 4.2.

The A-dependence of photoproduction on nuclei has been measured by several groups [46]. Figure 16 displays our results of the shadowing ratio A_{eff}/A calculated for C, Cu and Pb target nuclei, together with various experimental

data. We observe significant shadowing for $\nu > 2\,\mathrm{GeV}$. The effect is well described within our model. Its results are quite similar to those of earlier VMD calculations [13], which should not be much of a surprise, due to the dominant rôle of the VMD term noted above.

Shadowing grows stronger for higher photon energies ν, eventually approaching some saturation value, apart from logarithmic corrections due to inelastic contributions to multiple scattering (see discussion in Sections 4.2 and 4.3). This is due to the fact that the coherence length λ governing multiple scattering processes is now directly proportional to ν.

The validity of the picture developed here is restricted to $\nu > 2\,\mathrm{GeV}$. For smaller photon energies, photonuclear dynamics is governed by the excitation and propagation of nucleon resonances in nuclei.

6 Summary

Shadowing at small values $x < 0.1$ of the Bjorken variable is the most prominent nuclear effect seen in deep-inelastic lepton scattering from nuclear targets. We have developed a phenomenology of nuclear shadowing, expressed in the laboratory frame, which makes use of the full hadronic spectrum of virtual photons. Our framework unifies the vector meson dominance picture with the concept of color transparency applied to the quark-antiquark continuum part of the photon spectral function.

Our results are summarized as follows:

i) In our lab frame approach shadowing arises from the coherent multiple scattering of quark-antiquark fluctuations through the nuclear target. A satisfactory description of nuclear structure functions at small x is achieved for a large variety of nuclei, both light and heavy.

ii) Vector mesons dominate at small $Q^2 \lesssim 1\,\mathrm{GeV}^2$. At large Q^2 the quark-antiquark continuum becomes important. The combination of both resonant and continuum parts of the hadronic photon spectrum is crucial in order to obtain the almost negligible overall Q^2 dependence of the shadowing effect, whereas vector meson dominance alone would imply decreasing shadowing with increasing Q^2.

iii) Contributions to shadowing from inelastic intermediate states in the multiple scattering chain give only small corrections for heavy nuclei, but they need to be taken into account for light nuclei, in particular for the deuteron.

iv) Consistency is found with the observed shadowing for interactions of real photons with nuclei at high energies.

18

v) Special emphasis has been directed to shadowing effects in deuterium. In the extraction of the neutron structure function from deuteron data at small x, such effects must be taken into account carefully. We find strong sensitivity to the short distance behavior of the deuteron wave function; the use of realistic nucleon-nucleon potentials is therefore absolutely necessary for a reliable description. Shadowing effects of 1–2% imply corrections of the order of 10% in the Gottfried sum. This correction further increases the already established discrepancy with the naive parton model.

References

[1] NMC, P. Amaudruz et al., Z. Phys. C 51 (1991) 387;
NMC, P. Amaudruz et al., A re-evaluation of the nuclear structure function ratios for D, He, ^{6}Li, C and Ca, *submitted to Nucl. Phys B.*

[2] NMC, P. Amaudruz et al., Phys. Lett. B 295 (1992) 195.

[3] NMC, M. Arneodo et al., Phys. Rev. D 50 (1994) R1.

[4] A. Paić, Structure function ratios $F_2^{\mathrm{Li}}/F_2^{\mathrm{D}}$ and $F_2^{\mathrm{C}}/F_2^{\mathrm{D}}$ at low x_{bj}, in *PAN XIII, Particles and Nuclei, XIII international conference*, edited by A. Pascolini, Singapore, 1994, World Scientific;
NMC, M. Arneodo et al., The structure function ratios $F_2^{\mathrm{Li}}/F_2^{\mathrm{D}}$ and $F_2^{\mathrm{C}}/F_2^{\mathrm{D}}$ at small x, *submitted to Nucl. Phys. B.*

[5] FNAL E665, M. R. Adams et al., Phys. Rev. Lett. 68 (1992) 3266.

[6] FNAL E665, M. R. Adams et al., Phys. Lett. B 287 (1992) 375.

[7] FNAL E665, Timothy J. Carroll, Ratios of cross sections of carbon, calcium and lead at low x_{Bj} in inelastic muon scattering, FERMILAB-Conf-93/166-E, 1993, presented at the *28th Rencontres de Moriond, QCD and High Energy Hadronic Interactions*, Les Arcs, Savoie, France.

[8] FNAL E665, P. Spentzouris et al., Structure functions and structure function ratio F_2^n/F_2^p at small x_{bj} and Q^2 in muon-nucleon scattering, FERMILAB-Conf-94/220-E, 1994, presented at *Intersections of Particle and Nuclear Physics,* St. Petersburg, FL.

[9] B. Badełek and J. Kwieciński, Rev. Mod. Phys. 64 (1992) 927; M. Arneodo, Phys. Rep. 240 (1994) 301.

[10] A. H. Mueller and J. Qiu, Nucl. Phys. B 268 (1986) 427; J. Qiu, Nucl. Phys. B 291 (1987) 746.

[11] J. J. Sakurai and D. Schildknecht, Phys. Lett. B 40 (1972) 121; P. Ditsas, B. J. Read, and G. Shaw, Nucl. Phys. B 99 (1975) 85; C. L. Bilchak, D. Schildknecht, and J. D. Stroughair, Phys. Lett. B 214 (1988) 441, Phys. Lett. B 233 (1989) 461.

[12] G. Shaw, Phys. Lett. B 228 (1989) 125, Phys. Rev. D 47 (1993) 3676.

[13] T. H. Bauer, R. D. Spital, D. R. Yennie, and F. M. Pipkin, Rev. Mod. Phys. 50 (1978) 261.

[14] L. L. Frankfurt and M. I. Strikman, Nucl. Phys. B 316 (1989) 340.

[15] S. J. Brodsky and H. J. Lu, Phys. Rev. Lett. 64 (1990) 1342.

[16] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 49 (1991) 607.

[17] B. Badełek and J. Kwieciński, Nucl. Phys. B 370 (1992) 278.

[18] W. Melnitchouk and A. W. Thomas, Phys. Lett. B 317 (1993) 437.

[19] S. A. Kulagin, G. Piller, and W. Weise, Phys. Rev. C 50 (1994) 1154.

[20] G. Piller and W. Weise, Phys. Rev. C 42 (1990) R1834, Nucl. Phys. A 532 (1991) 271c.

[21] A. Donnachie and G. Shaw, Generalized vector dominance, in *Electromagnetic Interactions of Hadrons, Vol. 2*, edited by A. Donnachie and G. Shaw, page 169, Plenum Press, New York, 1978.

[22] N. N. Nikolaev, Comments on Nuclear and Particle Physics 21 (1992) 41; L. L. Frankfurt, G. A. Miller, and M. Strikman, Ann. Rev. of Nucl. and Part. Sci. 44 (1994) 501.

[23] S. Dasu et al., Phys. Rev. D 59 (1994) 5641.

[24] V. N. Gribov, JETP 30 (1970) 709.

[25] J. D. Bjorken and J. Kogut, Phys. Rev. D 8 (1973) 1341.

[26] PDG, L. Montanet et al., Phys. Rev. D 50 (1994).

[27] EMC, J. J. Aubert et al., Nucl. Phys. B 213 (1983) 1.

[28] S. D. Holmes, W. Lee, and J. E. Wiss, Ann. Rev. of Nucl. and Part. Sci. 35 (1985) 397.

[29] B. Z. Kopeliovich, J. Nemchick, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. B 309 (1993) 179.

[30] G. V. Davidenko and N. N. Nikolaev, Nucl. Phys. B 135 (1978) 333.

[31] L. Bertocchi, Nuovo Cim. 11A (1972) 45; J. H. Weis, Acta Physica Polonica B7 (1976) 851.

[32] P. V. R. Murthy, C. A. Ayre, H. R. Gustafson, L. W. Jones, and M. J. Longo, Nucl. Phys. B 92 (1975) 269.

[33] N. N. Nikolaev, Z. Phys. C 32 (1986) 537.

[34] W. Weise, Phys. Rep. 13 (1974) 53.

[35] H. de Vries, C. W. de Jager, and C. de Vries, Atomic Data and Nuclear Data Tables 36 (1987) 495.

[36] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B 260 (1991) 414.

[37] NMC, R. Seitz and A. Witzmann, The ratio F_2^n/F_2^p and the A-dependence of nuclear ratios measured by the NMC, 1993, Presented at the *28th Rencontres de Moriond, QCD and High Energy Hadronic Interactions*, Les Arcs, Savoie, France.

[38] N. N. Nikolaev and V. R. Zoller, Z. Phys. C 56 (1992) 623; V. R. Zoller, Z. Phys. C 54 (1992) 425; W. Melnitchouk and A. W. Thomas, Phys. Rev. D 47 (1993) 3783; V. Barone, M. Genovese, N. N. Nikolaev, and E. Predazzi, Phys. Lett. B 321 (1994) 137; B. Badełek and J. Kwieciński, Phys. Rev. D 50 (1994) R4.

[39] K. Goulianos, Phys. Rep. 101 (1983) 171.

[40] R. L. Cool, K. Goulianos, S. L. Segler, H. Sticker, and S. N. White, Phys. Rev. Lett. 47 (1981) 701; T. J. Chapin et al., Phys. Rev. D 31 (1985) 17.

[41] M. Lacombe, B. Loiseau, J. M. Richard, and R. V. Mau, Phys. Rev. C 21 (1980) 861.

[42] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. 149 (1987) 1.

[43] L. Hulthén and M. Sugawara, The two nucleon problem, in *Handbuch der Physik*, edited by S. Flügge, volume 39, Springer, Berlin, 1957.

[44] K. Gottfried, Phys. Rev. Lett. 18 (1967) 1174.

[45] S. D. Ellis and W. J. Stirling, Phys. Lett. B 256 (1991) 258; E. M. Henley and G. A. Miller, Phys. Lett. B 251 (1990) 453; W.-Y. P. Hwang, J. Speth, and G. E. Brown, Z. Phys. A 339 (1991) 383; S. Kumano and J. T. Londergan, Phys. Rev. D 46 (1992) 457; J. Levelt, P. J. Mulders, and A. W. Schreiber, Phys. Lett. B 263 (1991) 498; B.-Q. Ma, A. Schäfer, and W. Greiner, Phys. Rev. D 47 (1993) 51; W. Melnitchouk and A. W. Thomas, Phys. Rev. D 47 (1993) 3794; A. Szczurek and J. Speth, Nucl. Phys. A 555 (1993) 249.

[46] V. Heynen, H. Meyer, B. Naroska, and D. Notz, Phys. Lett. 34B (1971) 651; G. R. Brookes et al., Phys. Rev. D 8 (1973) 2826; S. J. Michalowski et al., Phys. Rev. Lett. 39 (1977) 737; D. O. Caldwell et al., Phys. Rev. D 7 (1973) 1362; D. O. Caldwell et al., Phys. Rev. Lett. 42 (1979) 553; E. A. Arakelyan et al., Phys. Lett. 79B (1978) 143; N. Bianchi et al., Phys. Lett. 325B (1994) 333.

Tables

V	$m_V/\,\mathrm{MeV}$ [26]	$g_V^2/4\pi$	$\sigma_{VN}/\,\mathrm{mb}$	
ρ	769.9±0.8	2.0	22–27	[13]
ω	781.9±0.1	23.1	25–27	[13]
ϕ	1019.41±0.01	13.2	9–12	[13]
J/ψ	3096.88±0.04	10.5	2.2 ± 0.7	[27]
ψ'	3686.0±0.1	30.6	~ 1.3	[28]

Table 1: Vector meson properties: masses, couplings to the photon, and total vector meson-nucleon cross sections. The coupling constants g_V are derived from the $V \to e^+ e^-$ decay widths [26].

	$\Delta I_G(0.004, 0.1)$		
$I_G^{\mathrm{exp.}}(0.004, 0.8)$ [3]	Bonn(1)	Bonn(2)	Paris
0.221 ± 0.021	-0.022	-0.039	-0.017

Table 2: Shadowing corrections ΔI_G for the Gottfried sum obtained for various deuteron wave functions compared to the experimental value $I_G^{\mathrm{exp.}}$. Here "Bonn(1)" refers to the one-boson exchange Bonn potential, "Bonn(2)" is the full potential including explicit two-pion exchange etc. [42]. The Paris potential is taken from ref. [41].

Figure captions

1. The two possible time orderings for the interaction of a (virtual) photon with a nucleon or nuclear target: (a) the photon hits a quark in the target, (b) the photon creates a $q\bar{q}$ pair that subsequently interacts with the target.

2. Nucleon structure function for small x plotted against Q^2. The solid line is the full result of our calculation. The contribution of vector mesons is indicated by the dashed line. We compare to NMC data from ref. [2].

3. Contribution $\mathcal{A}_h^{(n)}$ to the multiple scattering series: the hadronic projectile scatters from n nucleons inside the target nucleus.

4. Our results for shadowing in He, Li, C, and Ca compared to available experimental data [1, 4, 7]. The dashed curves show the shadowing caused by the vector mesons ρ, ω and ϕ only.

5. Shadowing in Xenon. Data are from the FNAL E-665 experiment [5, 6]. The vector meson contribution is shown by the dashed curve.

6. The slope $b = \mathrm{d}R/\mathrm{d}\ln Q^2$ indicating the Q^2 dependence of the shadowing ratio for He, Li, C, and Ca extracted by the NMC [1, 4] for various x-bins together with our results.

7. Q^2 dependence of the shadowing ratio Sn/C as predicted by our model, in the region to be covered by recent NMC data.

8. The shadowing ratio in Xe (a) as a function of x at fixed ν; (b) as a function of Q^2 at fixed ν.

9. The shadowing ratio as a function of the nuclear mass number A for several x-bins. Experimental data are preliminary NMC results from [37].

10. Our results for the slopes b_x of the A dependence relative to Carbon obtained from Eq. (30) compared to the NMC data in ref. [37].

11. Shadowing ratios in deuterium plotted against x at different values of Q^2 and for different deuteron wave functions. Here "Bonn(1)" refers to the one-boson exchange Bonn potential, "Bonn(2)" is the full potential including explicit two-pion exchange etc. [42]. The Paris potential is taken from ref. [41].

12. The density $\rho(r) = (u^2(r) + w^2(r))/4\pi r^2$ corresponding to different parametrizations of the deuteron wave function.

13. The shadowing ratio for deuterium at fixed $Q^2 = 4\,\mathrm{GeV}^2$ calculated for the Paris wave function. The full line includes inelastic intermediate states. The dashed curve was obtained by taking only elastic intermediate states into account.

14. Gottfried sum and shadowing: the filled symbols display the structure function difference $F_2^{p-n}(x)$, with the circles representing the original NMC data [3], squares and diamonds include shadowing correction using different deuteron wave functions. Open symbols show the respective values for the integral $I_G(x, 0.8)$.

15. Our result for $R^D(x, Q^2)$ compared to recent FNAL E-665 [8] and NMC data [3] for the ratio $F_2^d/2F_2^p$.

16. The shadowing ratio for the absorption of real photons on nuclei as calculated in our model and measured by various groups [46] plotted against the photon energy ν.

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig2-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig3-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig2-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig3-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2

This figure "fig2-3.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9504407v2